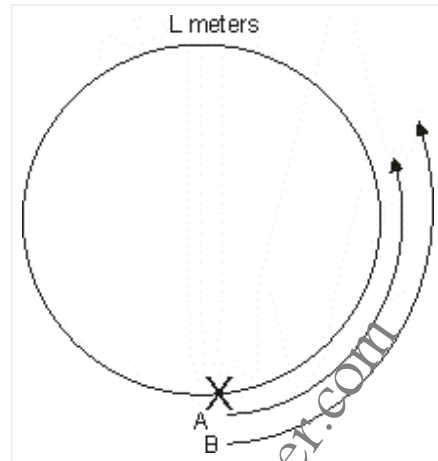


# Time Speed and Distance - 4 (Circular Tracks)

When two or three persons are running around a circular track, it is difficult to imagine the condition when they meet each other. But by thinking in a systematic order, we can solve these questions easily.

## CASE 1: Same direction

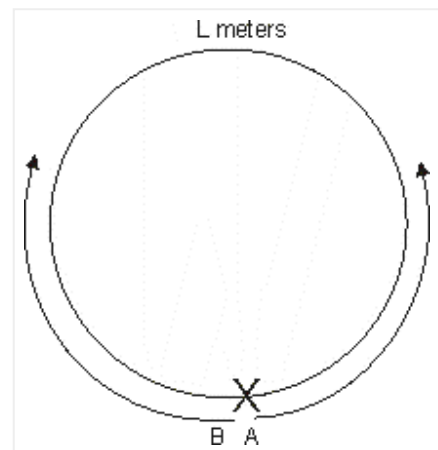


When two persons A, and B are running around a circular track of length L mts with speeds of a, b m/s in the same direction

They meet each other at any point on the track is  $\frac{L}{a-b}$  seconds

They meet each other at exactly at the starting point =  $\left(\frac{L}{a}, \frac{L}{b}\right)$  seconds

## CASE 2: opposite direction



When two persons A, and B are running around a circular track of length L mts with speeds of a, b m/s in opposite direction,

They meet each other at any point on the track is  $\frac{L}{a+b}$  seconds

They meet each other at exactly at the starting point =  $(\frac{L}{a}, \frac{L}{b})$  seconds

**CASE 3:** When three persons A, B and C are running around a circular track of length L mts with speeds of a, b and c m/s in the same direction direction,

They meet each other at any point on the track is  $\frac{L}{a-b}, \frac{L}{b-c}$  seconds

### Solved Examples:

1. In a circular race of 1200m, A and B start from the same point and at the same time with speeds of 27kmph and 45 kmph. Find when will they meet again for the first time on the trace when they are running in the same direction and Opposite direction?

a. 240, 60 Secs

b. 200, 60 secs

c. 200, 80 Sec

d. None of these

Sol: option A

Length of the track L=1200m

$$\text{Speed of A} = 27 \times \frac{5}{18} = 7.5 \text{ m/s}$$

$$\text{Speed of B} = 45 \times \frac{5}{18} = 12.5 \text{ m/s}$$

(i) same direction

$$\text{Time} = L / \text{Relative Speed} = \frac{1200}{12.5 - 7.5} = 240 \text{ sec}$$

(ii) Opposite Direction

$$\text{Time} = L / \text{Relative Speed} = \frac{1200}{12.5 + 7.5} = 60 \text{ sec}$$

2. In a circular race of 1200 m length, A and B start with speeds of 18kmph and 27 kmph starting at the same time from the same point. When will they meet for the first time at the starting point when running in the same direction and opposite direction?

a. 240, 160

b. 480, 480

c. 240, 240

d. 160, 480

Length of the track L = 1200 m

$$\text{Speed of A} = 18 \times \frac{5}{18} = 5 \text{ m/s}$$

$$\text{Speed of B} = 27 \times \frac{5}{18} = 7.5 \text{ m/s}$$

$$\text{Time taken by A to complete one round} = \frac{1200}{5} = 240 \text{ sec}$$

$$\text{Time take by B to complete one round} = \frac{1200}{7.5} = 160 \text{ sec}$$

(i) Same direction:

They will meet at the starting point at a time which is the LCM of the timings taken by each of them to complete one full round. i.e., the LCM of 160s and 240 s which is 480 Sec

(ii) Opposite Direction:

They will meet at the starting point at a time which is the LCM of the timings taken by each of them to complete one full round. i.e., the LCM of 160s and 240s which is 480 Sec

3. A, B and C run around a circular track of length 1200 m with respective speeds 9, 18, 27 kmph. If they started at the same time from the same point and run in the same direction when will they meet for the first time?

- a. 360 sec
- b. 480 sec
- c. 240 sec
- d. None

L = 1200 m

Speed of A (a) =  $9 \times \frac{5}{18} = 2.5$  m/sec

Speed of B (b) =  $18 \times \frac{5}{18} = 5$  m/sec

Speed of C (c) =  $27 \times \frac{5}{18} = 7.5$  m/sec

They will meet for the first time at a time which is the  $\frac{L}{a-b}, \frac{L}{b-c}$  seconds

$$\frac{L}{a-b} = \frac{1200}{5-2.5} = 480 \text{ sec}$$

$$\frac{L}{b-c} = \frac{1200}{7.5-5} = 480 \text{ sec}$$

So they will meet for the first time after 480 sec. i.e., 8 min after they start.

4. P, Q, R run around a circular track 1200 m long with speed of 9, 18, 27 kmph. If they start at the same point and at the same time in the same direction, when will they meet again at the starting point?

- a. 360 sec
- b. 480 sec
- c. 240 sec
- d. None

L = 1200 m

Speed of P(p) =  $9 \times \frac{5}{18} = 2.5$  m/sec

Speed of Q (q) =  $18 \times \frac{5}{18} = 5$  m/sec

Speed of R (r) =  $27 \times \frac{5}{18} = 7.5$  m/sec

They will meet for the first time at a time which is the LCM of  $L/p, L/a, L/r$

$$L/p = 1200 / 2.5 = 480$$

$$L/q = 1200 / 5 = 240$$

$$L/r = 1200 / 7.5 = 160$$

LCM of 480, 240, 160 is 480 Sec. So they meet after 8 min.

5. A can run one full round of a circular track in 6 min and B in 15 min. If both A and B start simultaneously from the same starting point then How many times would they met in the time B has completed 10 rounds when running in same direction, and In opposite direction?

- a. 15, 10
- b. 25, 30
- c. 25, 35
- d. None

When B has completed 10 rounds, A would have completed  $10 \times 15/6 = 25$  rounds.

When running in same direction, this would mean A having run 15 rounds more than B and would thus have met 15 times (For every one round that A runs more than B, A meets B)

When running in Opposite direction, this would mean A and B together having run 35 rounds and thus would have met 35 times.

6. Bhim and Arjun were exercising during their Vanvaas. They start running on a circular track simultaneously and in the same direction. If Bhim takes 4 min to complete one round, and Arjun takes 7 min to complete one round Find (i) After how much time will they meet for the first time (ii) After how much time will they meet for the first time at the starting point (iii) After how much time would they meet for the first time at a point diametrically opposite to the starting point on the track?

- a.  $14/3$ , 28, 28
- b.  $28/3$ , 28, they never meet
- c.  $28/3$ , 28, 14
- d. None

**Sol: Option C**

Ratio of speed of Bhim and Arjun = 7:4

a. If the length of circular track = 28 m, the speeds of Bhim and Arjun are 7 and 4 m/ min

The time when they are together for the first time will be when Bhim (the faster one) has taken one round more than Arjun.

Therefore, if time when they meet is 't' then

$7t - 4t = 28$ . which means  $t = 28/3$  min

b. They will meet at the starting place the first time at a time which is the LCM of the times each one of them takes to reach the starting place

Therefore, LCM of 4, 7 is 28 min.

C. Diametrically opposite point is at a circular distance of 14 m.

Bhim reaches this point in  $14/7 = 2$  min and Arjun reaches this point in  $14 / 4 = 3.5$  min.

Bhim reaches this point in the 2nd min,  $2+4 = 6$  min,  $6+4 = 10$ th min... so on. Arjun reaches after 3.5 min, 10.5 min, 17.5 min ...so on.

The time after the start when Bhim reaches the point is a natural number, Where as the time when Arjun reaches this point will always be a non-natural number. So they will never meet.

Alternatively:

If the time when they would meet for the first time at the starting point = LCM (4, 7) = 28 min, in this time Bhim does 7 rounds and Arjun completes 4 rounds. Hence, He would take (28/3) min to take a lead of one round. This is the time they would meet for the first time.

### Additional Questions:

7. Amar and Balu as a warm up exercise are jogging on a circular track. Balu is a better athlete and jogs at 18 km/hr, while Amar jogs at 9 km/hr. The circumference of the track is 500 m. They start from the same point and in the same direction. When will they be together again for the first time?

Since Balu is faster than Amar he will take a lead and as they keep running, the gap between them will also keep widening, unlike on a straight track they would meet again even if Balu is faster than Amar.

The same problem could be rephrased as: In what time would Balu take a lead of 500 m over Amar?

Every second, Balu is taking a lead of  $m = 2.5$  m over Amar.

Hence, he takes to take a lead of 500 m over Amar. Hence, they would meet for the first time after 200 s.

### Alternative method

For every round that Amar makes, Balu would have made 2 rounds because the ratio of their speeds is 1 : 2. Hence, when Amar has made one full round, Balu would have taken a lead of one round.

Therefore, they would meet after  $\frac{500}{2.5}$  s, i. e.  $\left[ \frac{1 \text{ round}}{\text{Amar's speed}} = \frac{500}{2.5} \right] = 200$  s

[Here,  $9 \times \frac{5}{18} = 2.5$  m/s is Amar's speed.]

8. If the speeds of Dhoni and Sachin were 8 km/hr and 5 km/hr, then after what time will the two meet for the first time at the starting point if they start simultaneously? The length of the circular track is 500 m.

Let us first calculate the time Dhoni and Sachin take to make one full circle.

$$\text{Time taken by Dhoni} = \frac{500}{\left(8 \times \frac{5}{18}\right)} = 225 \text{ s}$$

Hence, after every 225 s, Dhoni would be at the starting point and after every 360 s, Sachin would be at the starting point. The time, when they will be together again at the starting point simultaneously for the first time, would be the smallest multiple of both 225 and 360, which is the LCM of 225 and 360.

Hence, they would both be together at the starting point for the first time after LCM (225, 360) = 1800 s. Thus, every half an hour, they would meet at the starting point.

From the solution you could realize that it is immaterial whether they move in the same direction or in the opposite.

9. A and B walk around a circular path of 900 metre in circumference, starting together from the same point in the

same direction. If their speeds are 150 metre per minute and 200 metre per minute respectively, after how many minutes will they be again at the starting point?

The time after which they are together at the starting point = LCM of  $(\frac{L}{a}, \frac{L}{b})$  seconds

So A takes  $900 / 150 = 6$  min and B takes  $900 / 200 = 9/2$  min

LCM of  $6/1$  and  $9/2 = \text{LCM of } (6, 9) / \text{HCF of } (1, 2) = 18/1 = 18$  minutes.

[www.FirstRanker.com](http://www.FirstRanker.com)